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ON THE SUPERSTRINGS-INDUCED FOUR-DIMENSIONAL GRAVITY, AND ITS APPLICATIONS TO COSMOLOGY ¹

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Abstract

We review the status of the fourth-order (quartic in the spacetime curvature) terms induced by superstrings/M-theory (compactified on a warped torus) in the leading order with respect to the Regge slope parameter, and study their (non-perturbative) impact on the evolution of the Hubble scale in the context of the four-dimensional FRW cosmology. After taking into account the quantum ambiguities in the definition of the off-shell superstring effective action, we propose the generalized Friedmann equations, find the existence of their (de Sitter) exact inflationary solutions without a spacetime singularity, and constrain the ambiguities by demanding stability and the scale factor duality invariance of our solutions. The most naive (Bel-Robinson tensor squared) quartic terms are ruled out, thus giving the evidence for the necessity of extra quartic (Ricci tensor-dependent) terms in the off-shell gravitational effective action for superstrings. Our methods are generalizable to the higher orders in the spacetime curvature.

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1 Introduction

The homogeneity and isotropy of our Universe, as well as the observed spectrum of density perturbations, are explained by inflationary cosmology [1]. Inflation is usually realised by introducing a scalar field (inflaton) and choosing an appropriate scalar potential. When using Einstein equations, it gives rise to the massive violation of the strong energy condition and the exotic matter with large negative pressure. Despite of the apparent simplicity of such inflationary scenarios, the origin of their key ingredients, such as the inflaton and its scalar potential, remains obscure.

Theory of superstrings is the leading candidate for a unified theory of Nature, and it is also the only known consistent theory of quantum gravity. It is therefore natural to use superstrings or M-theory for the construction of specific mechanisms of inflation. Recently, many brane inflation scenarios were proposed (see e.g. ref. [2] for a review), together with their embeddings into the (warped) compactified superstring models, in a good package with the phenomenological constraints coming from particle physics (see e.g. ref. [3]). However, it did not contribute to revealing the origin of the key ingredients of inflation. It also greatly increased the number of possibilities up to 10^{500} (known as the String Landscape), hampering specific theoretical predictions in the search for the signatures of strings and branes in the Universe.

The inflaton driven by a scalar potential and their engineering by strings and branes are by no means required. Another possible approach can be based on a modification of the gravitational part of Einstein equations by terms of the higher order in the spacetime curvature [4]. It does not require an inflaton or an exotic matter, while the specific higher-curvature terms are well known to be present in the effective action of superstrings [5].

The perturbative strings are defined on-shell (in the form of quantum amplitudes), while they give rise to the infinitely many higher-curvature corrections to the Einstein equations, to all orders in the Regge slope parameter α' and the string coupling g_s . The finite form of all those corrections is unknown and beyond our control. However, it still makes sense to consider the *leading* corrections to the Einstein equations, coming from strings and branes. Of course, any results to be obtained from the merely leading quantum corrections cannot be conclusive. Nevertheless, they may offer both qualitative and technical insights into the early Universe cosmology, within the well defined and highly restrictive framework. In this paper we adopt the approach based on the Einstein equations modified by the leading

superstring-generated gravitational terms which are *quartic* in the spacetime curvature. We treat the quartic curvature terms *on equal footing* with the Einstein term, i.e. non-perturbatively.

We consider only geometrical (i.e. pure gravity) terms in the low-energy M-theory effective action in four space-time dimensions. We assume that the quantum g_s -corrections can be suppressed against the leading α' -corrections, whereas all the moduli, including a dilaton and an axion, are somehow stabilized (e.g. by fluxes, after the warped compactification to four dimensions and spontaneous supersymmetry breaking).

Our paper is organized as follows. In Sec. 2 we review our starting point: M-theory in 11 spacetime dimensions with the leading quantum corrections, and the dimensional reduction to four spacetime dimensions. In Sec. 3 we discuss the problem of the off-shell extension of the gravitational part of the four-dimensional effective action for superstrings. In Sec. 4 we review the physical significance of the on-shell quartic curvature terms. In Sec. 5 we prove that it is impossible to eliminate the 4th order time derivatives in the 4-dimensional equations of motion with a generic metric. The structure of equations of motions for the special (FRW) metrics is revealed in Sec. 6, which contains our main new results. The exact (de Sitter) solutions, stability and duality constraints are also discussed in Sec. 6. Our conclusion is Sec. 7. In Appendix A we give our notation and compute some relevant identities. The two-component spinor formalism (for completeness) is summarized in Appendix B.

2 M-theory and modified Einstein equations

There are five perturbatively consistent superstring models in ten spacetime dimensions (see e.g. the book [5]). All those models are related by duality transformations. In this paper we are going to consider only the gravitational sector of the heterotic and type-II strings. In addition, there exists a parent theory behind all those superstring models, it is called M-theory, and it is eleven-dimensional [5]. Not so much is known about the non-perturbative M-theory. Nevertheless, there are the well-established facts that (i) the M-theory low-energy effective action is given by the 11-dimensional supergravity [6], and (ii) the leading quantum gravitational corrections to the 11-dimensional supergravity from M-theory in the bosonic sector are quartic in the curvature [7, 8] (see e.g. ref. [9] for some recent progress). Our purpose

in this Section is to emphasize what is not known.

All the bosonic terms of the M-theory corrected 11-dimensional action read as follows [7, 8]:

$$S_{11} = -\frac{1}{2\kappa_{11}^2} \int d^{11}x \sqrt{-g} \left[R - \frac{1}{2 \cdot 4!} F^2 - \frac{1}{6 \cdot 3! \cdot (4!)^2} \varepsilon_{11} C F F \right] \\ - \frac{T_2}{(2\pi)^4 \cdot 3^2 \cdot 2^{13}} \int d^{11}x \sqrt{-g} \left(J - \frac{1}{2} E_8 \right) + T_2 \int C \wedge X_8 \quad (2.1)$$

where κ_{11} is the 11-dimensional gravitational constant, T_2 is the M2-brane tension given by

$$T_2 = \left(\frac{2\pi^2}{\kappa_{11}^2} \right)^{1/3}, \quad (2.2)$$

C is a 3-form gauge field of the 11-dimensional supergravity [6], and $F = dC$ is its four-form field strength, R is the gravitational scalar curvature, ε_{11} stands for the 11-dimensional Levi-Civita symbol in the Chern-Simons-like coupling, while (J, E_8, X_8) are certain *quartic* polynomials in the 11-dimensional curvature. The J is given by

$$J = 3 \cdot 2^8 \left(R^{mijn} R_{pijq} R_m{}^{rsp} R^q{}_{rsn} + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R^q{}_{rsn} \right) + \mathcal{O}(R_{mn}), \quad (2.3)$$

the E_8 is the 11-dimensional extension of the eight-dimensional Euler density,

$$E_8 = \frac{1}{3!} \varepsilon^{abcm_1 n_1 \dots m_4 n_4} \varepsilon_{abcm'_1 n'_1 \dots m'_4 n'_4} R^{m'_1 n'_1}{}_{m_1 n_1} \dots R^{m'_4 n'_4}{}_{m_4 n_4} \quad (2.4)$$

and the X_8 is the eight-form

$$X_8 = \frac{1}{192 \cdot (2\pi^2)^4} \left[\text{tr} R^4 - \frac{1}{4} (\text{tr} R^2)^2 \right], \quad (2.5)$$

where the traces are taken with respect to (implicit) Lorentz indices in eleven space-time dimensions. The (world) vector indices are also suppressed in eq. (2.1).

The J -contribution (2.3) is defined *modulo* Ricci-dependent terms by its derivation [7, 8]. The basic reason is the *on-shell* nature of the perturbative superstrings [5], whose quantum on-shell amplitudes determine the gravitational effective action modulo field redefinitions. Via the Einstein-Hilbert

term, the metric field redefinitions contribute to the next (quartic) curvature terms with at least one factor of Ricci curvature. Therefore, some additional physical requirements are needed in order to fix those Ricci-dependent terms in the off-shell M-theory effective action.

To match the constraints imposed by particle physics, M-theory is supposed to be compactified to one of the superstring models in ten dimensions, and then down to four spacetime dimensions e.g., on a Calabi-Yau complex three-fold [5]. Alternatively, M-theory may be directly compactified down to four real dimensions on a 7-dimensional special (G_2) holonomy manifold [10]. The bosonic fields of the action (2.1) are just an eleven-dimensional metric and a 3-form (there is no dilaton in eleven dimensions). In other words, the 11-dimensional action (2.1) is the most general starting point to discuss the M-theory/superstrings compactification.

In the presence of fluxes, we should consider the *warped* compactification, whose metric is of the form [11]

$$ds_{11}^2 = e^{2A(y)} ds_{\text{FRW}}^2 + e^{-2A(y)} ds_7^2 \quad , \quad (2.6)$$

where ds_{FRW}^2 is the FRW metric in (uncompactified) four-dimensional space-time (see eq. (5.1) below), ds_7^2 is a metric in compactified seven dimensions with the coordinates y^a , $a = 4, 5, 6, 7, 8, 9, 10$, and $A(y)$ is called a warp factor.

Since we are interested in the gravitational sector of the four-dimensional type-II superstrings, an explicit form of the 7-metric ds_7^2 is not needed. In the case of heterotic strings, one has to include the ‘anomalous’ term quadratic in the curvature (see below). We put all the four-dimensional scalars (like a dilaton, an axion and moduli) into the matter stress-energy tensor (in Einstein frame), and assume that they are somehow stabilized to certain fixed values. In addition, we do not consider any M-theory/superstrings solitons such as M- or D-branes. After dimensional reduction, the only gravitational terms coming from type-II superstrings in four dimensions are given by

$$S_4 = -\frac{1}{2\kappa^2} \int d^4x \sqrt{-g} (R + \beta J_R) \quad (2.7)$$

where we have introduced the Einstein coupling κ in four dimensions, and the four-dimensional counterpart J_R of J in eq. (2.3), $i, j = 0, 1, 2, 3$,

$$J_R = R^{mijn} R_{pijq} R_m{}^{rsp} R_{rsn}^q + \frac{1}{2} R^{mnij} R_{pqij} R_m{}^{rsp} R_{rsn}^q + O(R_{mn}) \quad (2.8)$$

The relation between the coupling constants κ_{11} and κ is given by

$$\kappa^2 = e^{5A} M_{\text{KK}}^7 \kappa_{11}^2 \quad (2.9)$$

where we have introduced the *Kaluza-Klein* (KK) compactification scale $M_{\text{KK}}^{-7} = \text{Vol}_7 \equiv \int d^7 y \sqrt{g_7}$ and the *average* warp factor A (with an integer weight p),

$$e^{pA} = \frac{1}{\text{Vol}_7} \int d^7 y \sqrt{g_7} e^{pA(y)} \quad (2.10)$$

We also find

$$\beta = \frac{1}{3} \left(\frac{\kappa^2}{2^{23/2} \pi^5 e^{14A} M_{\text{KK}}^7} \right)^{2/3} \quad (2.11)$$

of mass dimension -6 . For instance, when substituting the Planck scale $\kappa \approx 10^{-33} \text{cm}$ and $M_{\text{KK}}^{-1} \approx 10^{-15} \text{cm}$, and ignoring the warp factor, $A = 0$, we get the incredibly small (and, in fact, unacceptable – see Sec. 6) value

$$\beta \approx 10^{-118} \text{ cm}^6 \quad (2.12)$$

As regards the four-dimensional heterotic strings, the action (2.7) is to be supplemented by the term [12]

$$S_H = -\frac{1}{2\kappa^2} \int d^4 x \sqrt{-g} \left(\frac{1}{8} J_H \right) \quad (2.13)$$

where

$$J_H = R_{ijkl} R^{ijkl} + \mathcal{O}(R_{mn}) \quad (2.14)$$

again modulo Ricci-dependent terms.

The gravitational action is to be added to a matter action, which lead to the *modified* Einstein equations of motion (in the type II case, for definiteness)

$$R_{ij} - \frac{1}{2} g_{ij} R + \beta \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} (\sqrt{-g} J_R) = \kappa^2 T_{ij} \quad (2.15)$$

where T_{ij} stands for the energy-momentum tensor of all the matter fields (including dilaton and axion).

Due to the ambiguities in the definition of the J_R -polynomial, it is also possible to replace it by

$$J_C = C^{mijn} C_{pijq} C_m{}^{rsp} C_q{}_{rsn} + \frac{1}{2} C^{mnij} C_{pqij} C_m{}^{rsp} C_q{}_{rsn} + \mathcal{O}(R_{mn}) \quad (2.16)$$

where we have introduced the Weyl tensor in four dimensions [13], which is the traceless part of the curvature tensor – See Appendices A and B.

3 Going off-shell with the curvature terms

There are about 10^2 Ricci-dependent terms in the most general off-shell gravitational effective action that is *quartic* in the curvature. It also means about 100 new coefficients, which makes the fixing of the off-shell action to be extremely difficult. The quartic curvature terms are thus different from the *quadratic* curvature terms, present in the on-shell heterotic string effective action (2.13), whose off-shell extension is very simple (see below). It is, therefore, desirable to formulate some necessary conditions that any off-shell extension has to satisfy.

(i) The first condition is, of course, the vanishing of all extra terms (i.e. beyond those in eq. (2.8)) in the Ricci-flat case [14]. The perturbative superstring effective action is usually deducted from the superstring amplitudes, whose on-shell condition is just the Ricci-flatness. In the alternative method, known as the non-linear sigma-model beta-function approach, the Ricci-dependent ambiguities in the effective equations of motion (associated with the vanishing sigma-model beta-functions) arise via the dependence of the renormalization group beta-functions of the non-linear sigma-model upon the renormalization prescription, starting from two loops (see e.g. ref. [15] for details).

(ii) Supersymmetry requires all quantum bosonic corrections to be extendable to locally supersymmetric invariants. It can be made manifest in four spacetime dimensions, where the off-shell superspace formalism of $N = 1$ supergravity is available [16]. The Weyl tensor, Ricci tensor and scalar curvature belong to three different $N = 1$ superfields called $W_{\alpha\beta\gamma}$, $G_{\alpha\dot{\alpha}}$ and R , respectively, while the first superfield is chiral.⁴ In particular, the Weyl tensor $C_{\alpha\beta\gamma\delta}$ appears in the first order of the $N = 1$ superspace chiral anti-commuting coordinates θ^α as

$$W_{\alpha\beta\gamma}(x, \theta) = W_{\alpha\beta\gamma}(x) + \theta^\delta C_{\alpha\beta\gamma\delta}(x) + \dots \quad (3.1)$$

so that the J_H terms (with all curvatures being replaced by Weyl tensors) is easily supersymmetrizable in superspace as

$$\int d^2\theta \mathcal{E}^{-1} W^2_{\alpha\beta\gamma} \quad (3.2)$$

⁴We use the two-component spinor notation [16], $\alpha, \beta, \dots = 1, 2$ — see Appendix B.

The J_C terms in eq. (2.16) are also extendable to the manifest superinvariant

$$\int d^4\theta E^{-1} W^2_{\alpha\beta\gamma} \overline{W}^2_{\dot{\alpha}\dot{\beta}\dot{\gamma}} \quad (3.3)$$

where we have introduced the supervielbein densities \mathcal{E} and E , in the chiral and central superspaces, respectively (see ref. [16] for details).

Those invariants were extensively studied in the past, because they naturally appear as the possible counterterms (with divergent coefficients) in quantum four-dimensional supergravity (see e.g. ref. [17]). In superstring theory one gets the *same* structures, though with *finite* coefficients (see e.g. refs. [18, 19]). Thus, in four dimensions, the structure of the *on-shell superstrings* quartic curvature terms is fixed by local $N = 1$ supersymmetry alone, up to normalization.

(iii) The absence of the higher order time derivatives is usually desirable to prevent possible unphysical solutions to the equations of motion, as well as preserve the perturbative unitarity, but it is by no means necessary. As is well known, the standard Friedmann equation of General Relativity is an evolution equation, i.e. it contains only the first-order time derivatives of the scale factor [1, 20]. It happens due to the cancellation of terms with the second-order time derivatives in the mixed 00-component of Einstein tensor — see e.g. Appendix of ref. [21] for details. It can also be seen as the consequence of the fact that the second-order dynamical (Raychaudhuri) equation for the scale factor in General Relativity can be integrated once, by the use of the continuity equation (3.5), thus leading to the evolution (Friedmann) equation [1]. As regards the quadratic curvature terms present in the heterotic case, their unique off-shell extension is given by the Gauss-Bonnet-type combination [22]

$$J_H \rightarrow G = R_{ijkl} R^{ijkl} - 4R_{ij} R^{ij} + R^2 \quad (3.4)$$

In the expansion around Minkowski space, $g_{ij}(x) = \eta_{ij} + h_{ij}(x)$, the fourth-order derivatives (at the leading order in $\mathcal{O}(h^2)$) coming from the first term in eq. (3.4) cancel against those in the second and third terms [23]. As a result, the off-shell extension (3.4) appears to be ghost-free in all dimensions. As regards *four* space-time dimensions, the terms (3.4) can be rewritten as the four-dimensional Euler density (8.7). Therefore, being a total derivative, eq. (3.4) does not contribute to the four-dimensional effective action.⁵

⁵Of course, adding Euler densities to the Einstein-Hilbert term matters in higher (than four) dimensions [24, 25], or with the dynamical dilaton and axion fields [26].

The higher time derivatives are apparent in the gravitational equations of motion with the quartic curvature terms (see also ref. [27]). It is natural to exploit the freedom of the metric field redefinitions, in order to get rid of those terms. However, in Sec. 5 we prove that it is impossible to eliminate the 4th order time derivatives in the quartic curvature terms via a metric field redefinition. It may still be possible for some special (like FRW) metrics, after imposing the string duality requirement (Sec. 6).

(iv) The matter equations of motion in General Relativity imply the covariant conservation law of the matter energy-momentum tensor,

$$(T^{ij})_{;j} = 0 \quad (3.5)$$

By the well known identity $(R^{ij} - \frac{1}{2}g^{ij}R)_{;j} = 0$, eqs. (2.15) and (3.5) imply

$$\left[\frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} (\sqrt{-g}J) \right]_{;j} = 0 \quad (3.6)$$

For instance, when $J = G$ as in eq. (3.4), eq. (3.6) reads

$$\begin{aligned} & -\frac{1}{2}(R_{ijkl}R^{ijkl} - 4R_{ij}R^{ij} + R^2)_{;m} + 2(R_{mjkl}R^{njlkl})_{;n} \\ & -4(R_{minj}R^{ij})_{;n} - 4(R_{mi}R^{in})_{;n} + 2(RR_{mn})^{;n} = 0 \end{aligned} \quad (3.7)$$

By the use of Bianchi identities for the curvature tensor, we find by an explicit calculation that the left-hand-side of eq. (3.7) identically vanishes. We believe that eq. (3.6) should be *identically* satisfied by *any* off-shell gravitational correction J because, otherwise, the consistency of the gravitational equations of motion may be violated.

Given the quartic curvature terms (2.8), the modified Einstein equations of motion (2.15) are

$$\begin{aligned} \kappa^2 T_{ij} = & R_{ij} - \frac{1}{2}g_{ij}R + \beta \left[-\frac{1}{2}g_{ij}J_R - R_{mhk(i}R_{j)rt}{}^m (R^{kqsr}R^t{}_{qs}{}^h + R^{ksqt}R^{hr}{}_{qs}) \right. \\ & - R_{kqs(i}R_{j)rmt} (R^{hsqt}R^{krm}{}_k - R^{thsq}R_h{}^{rmk}) + (R_{itrj}R^{ksqt}R^h{}_{sq}{}^r)_{(i;k;h)} \\ & \left. + (R_{isqt}R^{rktm}R_j{}^{sq}{}_k)_{(i;r;m)} - (R^{hrs(i}R_{j)mnr}R_h{}^{mnk} + R^{sht(i}R_{j)mnl}R^{kmn}{}_h)_{(i;k;s)} \right] \end{aligned} \quad (3.8)$$

(v) We may also add the *causality* constraint as our next condition: the group velocity of ultra-violet perturbations on a gravitational background

with the higher-curvature terms included, must not exceed the speed of light. As was demonstrated in ref. [28], the causality condition merely affects the sign factors of the full curvature terms in the action, namely, the signs in front of $(R_{mnpq}R^{mnpq})^2$ and $(R_{mnpq}^*R^{mnpq})^2$ should be positive. It must be automatically satisfied by the perturbative superstring quartic corrections (2.8) due to the known unitarity of superstring theory, and it is the case indeed — see the identity (8.21) — just because $\beta > 0$.

Of course, our list is not complete, and it could be easily extended by more conditions, e.g. by requiring the consistency with black hole physics, gravitational waves, nucleosynthesis, etc. For example, in Sec. 6 we impose the scale factor duality as yet another constraint.

4 On-shell structure and physical meaning of the quartic curvature terms

The detailed structure and physical meaning of the quartic curvature terms in eqs. (2.8) and (2.16) are easily revealed via their connection to the four-dimensional *Bel-Robinson* (BR) tensor [29]. The latter is well known in General Relativity [30, 31]. We review here the main properties of the BR tensor, and calculate the coefficients in the important identities – see eqs. (4.4) and (4.5) in this Section below.⁶

The BR tensor is defined by⁷

$$T_R^{iklm} = R^{ipql}R^k{}_{pq}{}^m + {}^*R^{ipql}{}^*R^k{}_{pq}{}^m \quad (4.1)$$

whose structure is quite similar to that of the Maxwell stress-energy tensor,

$$T_{ij}^{\text{Maxwell}} = F_{ik}F_j{}^k + {}^*F_{ik}{}^*F_j{}^k, \quad F_{ij} = \partial_i A_j - \partial_j A_i \quad (4.2)$$

The Weyl cousin T_C^{ijklm} of the BR tensor is obtained by replacing all curvatures by Weyl tensors in eq. (4.1)— see eq. (8.10). The Weyl BR tensor can be factorized in the two-component formalism (see Appendix B),

$$(T_C)_{\alpha\beta\gamma\delta\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = C_{\alpha\beta\gamma\delta}\bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \quad (4.3)$$

⁶Those coefficients were left undetermined in ref. [31].

⁷See also Appendix A for more.

In this section, we consider all the quartic terms on-shell, i.e. *modulo* Ricci-tensor dependent terms. Therefore, we are not going to distinguish between T_R and T_C here. The Ricci-tensor dependent additions will be discussed in Secs. 5 and 6.

The significance of the BR tensor to the quartic curvature terms is already obvious from superspace (see Sec. 3), where the locally N=1 supersymmetric extension of the quartic Weyl terms (2.16) is given by eq. (3.3) whose bosonic part is the BR tensor squared, due to eq. (4.3). As regards a straightforward proof, see Appendix A and our derivation of eq. (8.21) there, which imply

$$T_{ijkl}^2 = 8J_R = \frac{1}{4}(R_{ijkl}R^{ijkl})^2 + \frac{1}{4}(*R_{ijkl}R^{ijkl})^2 \quad (4.4)$$

In addition, when using another identity (8.18), eq. (4.4) yields

$$\begin{aligned} T_{ijkl}^2 &= 8J_R = -\frac{1}{4}(*R_{ijkl}^2)^2 + \frac{1}{4}(*R_{ijkl}R^{ijkl})^2 \\ &= \frac{1}{4}(P_4^2 - E_4^2) = \frac{1}{4}(P_4 + E_4)(P_4 - E_4) \end{aligned} \quad (4.5)$$

where we have introduced the Euler and Pontryagin topological densities in four dimensions — see eqs. (8.7) and (8.8), respectively.

In addition [29, 31], the on-shell BR tensor is *fully symmetric* with respect to its vector indices, it is *traceless*,

$$T_{ijkl} = T_{(ijkl)} \quad , \quad T_{ikl}^i = 0 \quad , \quad (4.6)$$

(ii) it is covariantly *conserved* (though the BR tensor is not a physical current!),

$$\nabla^i T_{ijkl} = 0 \quad , \quad (4.7)$$

and it has *positive* ‘energy’ density,

$$T_{0000} > 0 \quad . \quad (4.8)$$

Equation (4.6) is most easily seen in the two-component formalism (see Appendix B), eq. (4.7) is the consequence of Bianchi identities [32], whereas eq. (4.8) just follows from the definition (4.1).

The BR tensor is related to the gravitational energy-momentum *pseudo*-tensors [31]. It can be most clearly seen in *Riemann Normal Coordinates* (RNC) at any *given* point in spacetime. The RNC are defined by the relations

$$g_{ij} = \eta_{ij} \quad , \quad g_{ij,k} = 0 \quad , \quad g_{ij,mn} = -\frac{1}{3}(R_{imjn} + R_{injm}) \quad (4.9)$$

so that the derivatives of Christoffel symbols read as follows:

$$\Gamma_{jk,l}^i = -\frac{1}{3}(R_{jkl}^i + R_{kjl}^i) \quad (4.10)$$

Raising and lowering of vector indices in RNC are performed with Minkowski metric η_{ij} and its inverse η^{ij} , whereas all traces in the last two eqs. (4.9) and (4.10) vanish,

$$\eta^{ij}g_{ij,mn} = \eta^{ij}\Gamma_{ij,l}^k = \Gamma_{ij,k}^i = \Gamma_{jk,i}^i = 0 \quad (4.11)$$

Moreover, there exists the remarkable non-covariant relation (valid only in RNC) [31]

$$T_{ijkl} = \partial_k \partial_l (t_{ij}^{LL} + \frac{1}{2}t_{ij}^E) \quad (4.12)$$

where the symmetric *Landau-Lifshitz* (LL) gravitational pseudo-tensor [20]

$$\begin{aligned} (t_{LL})^{ij} = & -\eta^{ip}\eta^{jq}\Gamma_{pm}^k\Gamma_{qk}^m + \Gamma_{mn}^i\Gamma_{pq}^j\eta^{mp}\eta^{nq} - (\Gamma_{np}^m\Gamma_{mq}^j\eta^{in}\eta^{pq} + \Gamma_{np}^m\Gamma_{mq}^i\eta^{jn}\eta^{pq}) \\ & + h^{ij}\Gamma_{np}^m\Gamma_{mq}^n\eta^{pq} \end{aligned} \quad (4.13)$$

and the non-symmetric *Einstein* (E) gravitational pseudo-tensor [33]

$$(t^E)_j^i = (-2\Gamma_{mp}^i\Gamma_{jq}^m + \delta_j^i\Gamma_{pm}^n\Gamma_{qn}^m)\eta^{pq} \quad (4.14)$$

have been introduced in RNC, in terms of Christoffel symbols.

5 Off-shell quartic curvatures in cosmology

The main Cosmological Principle of a *spatially* homogeneous and isotropic (1 + 3)-dimensional universe (at large scales) gives rise to the standard *Friedman-Robertson-Walker* (FRW) metrics of the form [33]

$$ds_{\text{FRW}}^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right] \quad (5.1)$$

where the function $a(t)$ is known as the scale factor in ‘cosmic’ coordinates (t, r, θ, ϕ) ; we use $c = 1$ and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$, while k is the FRW topology index taking values $(-1, 0, +1)$. Accordingly, the FRW metric (5.1) admits a 6-dimensional isometry group G that is either $SO(1, 3)$, $E(3)$ or $SO(4)$, acting on the orbits $G/SO(3)$, with the spatial 3-dimensional sections

H^3 , E^3 or S^3 , respectively. By the coordinate change, $dt = a(t)d\eta$, the FRW metric (5.1) can be rewritten to the form

$$ds^2 = a^2(\eta) \left[d\eta^2 - \frac{dr^2}{1 - kr^2} - r^2 d\Omega^2 \right] \quad (5.2)$$

which is manifestly (4-dim) conformally flat in the case of $k = 0$. Therefore, the 4-dim Weyl tensor of the FRW metric obviously vanishes in the ‘flat’ case of $k = 0$. It is well known that the FRW Weyl tensor vanishes in the other two cases, $k = -1$ and $k = +1$, too [34, 21]. Thus we have

$$C_{ijkl}^{\text{FRW}} = 0 \quad (5.3)$$

Inflation in an early universe is defined as the epoch during which the scale factor is accelerating [1],

$$\ddot{a}(t) > 0, \text{ or equivalently } \frac{d}{dt} \left(\frac{H^{-1}}{a} \right) < 0 \quad (5.4)$$

where the dots denote time derivatives, and $H = \dot{a}/a$ is Hubble ‘constant’. The amount of inflation is given by a number of e-foldings [1],

$$N = \ln \frac{a(t_{\text{end}})}{a(t_{\text{start}})} = \int_{t_{\text{start}}}^{t_{\text{end}}} H dt \quad (5.5)$$

which should be around 70 [1].

Though the leading purely geometrical (perturbative) correction in the heterotic string case is given by the Gauss-Bonnet combination (3.4), and thus it does not contribute to the equations of motion in four space-time dimensions, the situation changes when the *dynamical* moduli (axion and dilaton) are included. The effective string theory couplings are moduli-dependent, which gives rise to a non-trivial coupling with the moduli in front of the Gauss-Bonnet term, so that the latter is not a total derivative any more. At the level of the one-loop corrected heterotic superstring effective action in four dimensions, the cosmological solutions were studied in ref. [26]. As regards the realization of inflation in M-theory, see e.g. ref. [35].

In the case of type-II superstrings (after stabilizing the moduli) we are left with the quartic curvature terms in the four-dimensional effective action (Sec. 2). Let’s address the issue of the higher time derivatives in the general

setting. It is quite natural to use the freedom of the metric field redefinitions in string theory in order to try to get rid of the higher time derivatives in the effective action. The successful example is provided by the Gauss-Bonnet gravity (Sec. 3) that we are now going to follow. Let's consider a weak gravitational field ⁸

$$g_{ij}(x) = \eta_{ij} + h_{ij}(x) \quad (5.6)$$

in the harmonic gauge

$$(h_{ij})^{;j} = \frac{1}{2}\partial_i h, \quad h = \eta^{ij}h_{ij} \quad (5.7)$$

The linearized curvatures are given by

$$R_{ijkl} = \frac{1}{2} [h_{il,jk} - h_{jl,ik} - h_{ik,jl} + h_{jk,il}] \quad (5.8)$$

whereas the Ricci tensor and the scalar curvature in the gauge (5.7) read

$$R_{ij} = -\frac{1}{2}\square h_{ij}, \quad R = -\frac{1}{2}\square h, \quad \square \equiv \partial^i \partial_i \quad (5.9)$$

As is clear from the structure of those equations, it is possible to form the Ricci terms after integration by parts in the quadratic curvature action. As a result, there is a cancellation of all terms with the 4th order time derivatives in the leading order $\mathcal{O}(h^2)$ of the Gauss-Bonnet action (3.4) in all spacetime dimensions, as was first observed in ref. [23].

Unfortunately, we find that it does not work for the quartic curvature terms, even in four spacetime dimensions, as we now going to argue.

When using the linearized curvature (5.8), the quartic terms (2.8) in four spacetime dimensions have the structure

$$\begin{aligned} 2^5 J_R = & A^{ikjl} A_{iljk} + 2A^{ikjl} B_{iljk} + B^{ikjl} B_{iljk} \\ & + A^{ikjl} \{C_{ilkj} + C_{lij k}\} + B^{ikjl} \{C_{ilkj} + C_{lij k}\} \\ & + 2C^{ikjl} \{C_{ilkj} + C_{lij k} + C_{kjil} + C_{jkli}\} \\ & - C^{ikjl} \{C_{iljk} + C_{likj} + C_{jkil} + C_{kjli}\} \end{aligned} \quad (5.10)$$

⁸We assign the lower case *latin* letters to spacetime indices, $i, j, k, \dots = 0, 1, 2, 3$, and the lower case *middle greek* letters to spatial indices, $\mu, \nu, \dots = 1, 2, 3$.

where we have introduced the notation $\partial_{ij}^2 = \partial_i \partial_j$ and

$$\begin{aligned} A^{ikjl} &= \partial_{mn}^2 h^{ik} (\partial^{2mn} h^{jl} + \partial^{2jl} h^{mn} - \partial^{2jm} h^{ln} - \partial^{2ln} h^{jm}), \\ B^{ikjl} &= \partial^{2ik} h_{mn} (\partial^{2mn} h^{jl} + \partial^{2jl} h^{mn} - \partial^{2jm} h^{ln} - \partial^{2ln} h^{jm}), \\ C^{ikjl} &= \partial_m^{2i} h_n^k (\partial^{2mn} h^{jl} + \partial^{2jl} h^{mn} - \partial^{2jm} h^{ln} - \partial^{2ln} h^{jm}) \end{aligned} \quad (5.11)$$

while all the index contractions above are performed with Minkowski metric.

Equation (5.10) is not very illuminating, but it is enough to observe that the dangerous terms $(\partial_{00}^2 h_{\mu\nu})^4$ and $(\partial_0 \partial_\lambda h_{\mu\nu})(\partial_{00}^2 h_{\mu\nu})^3$ do contribute, and thus lead to the terms with the 4th and 3rd order time derivatives in the equations of motion, when all $h_{\mu\nu}$ are supposed to be independent. The last possibility is to convert those terms into some Ricci-tensor dependent contributions. However, in the harmonic gauge (5.7), getting the Ricci tensor requires the two spacetime derivatives to be contracted into the wave operator, as in eq. (5.9), in each dangerous term, which is impossible for the quartic curvature terms, unlike their quadratic counterpart, because any integration by parts in the quartic terms does not end up with a wave operator in each term. The equations of motion in the case of $(BR)^2$ -gravity with the FRW metric are explicitly computed in the next Sec. 6, as an example.

Having failed to remove the higher time derivatives for a generic metric, one can try to get rid of them for a special class of metrics, namely, the FRW metrics of our interest. The simplest example arises when all the Riemann curvatures in the quartic curvature terms are replaced by the Weyl tensors, as in eq. (2.16). It also amounts to adding certain quartic curvature terms with at least one Ricci factor to the effective action (2.7). This proposal is based on the reasonable assumption [36] coming from the AdS/CFT correspondence that the $AdS_7 \times S^4$ and $AdS_4 \times S^7$ spaces seem to be the exact solutions to the (eleven-dimensional) M-theory equations of motion. Of course, such assumption is just the sufficient condition, not the necessary one, because there may be many more solutions. The substitution $R_{ijkl} \rightarrow C_{ijkl}$ leads to the contributions with three Weyl tensors (from the quartic terms) in the equations of motion, which implies *no* perturbative superstring corrections to the FRW metrics at all, because of eq. (5.3).

In the next Sec. 6 we find that the scale factor duality requirements allow a family of the generalized Friedmann equations coming from the most general quartic curvature terms, with just a few real parameters.

6 Exact solutions, stability and duality

Our motivation in this paper is based on the observation that the Standard Model (SM) of elementary particles does not have an inflaton.⁹ In addition, M-theory/superstrings have plenty of inflaton candidates but any inflationary mechanism based on a scalar field is highly model-dependent. When one wants the universal geometrical mechanism of inflation based on gravity only, it should occur due to some Planck scale physics to be described by the higher curvature terms (cf. ref. [4]).

On the experimental side, it is known that the vacuum energy density ρ_{inf} during inflation is bounded from above by a (non)observation of tensor fluctuations of the Cosmic Microwave Background (CMB) radiation [38],

$$\rho_{\text{inf}} \leq (10^{-3} M_{\text{Pl}})^4 \quad (6.1)$$

It severely constrains but does not exclude the possibility of the geometrical inflation originating from the purely gravitational sector of string theory, because the factor of 10^{-3} above may be just due to some numerical coefficients (cf. Sec. 2).

In this Section we consider the structure of our generalized Friedmann equation with *generic* quartic curvature terms. We get the conditions of stability of our inflationary solutions, and solve the duality invariance constraints coming from string theory [39].

Due to a single arbitrary function $a(t)$ in the FRW Ansatz (5.1), it is enough to take only one gravitational equation of motion in eq. (2.15) without matter, namely, its mixed 00-component. As is well known [1], the spatial (3-dimensional) curvature can be ignored in a very early universe, so we choose the manifestly conformally-flat FRW metric (5.1) with $k = 0$ in our Ansatz. It leads to a purely gravitational equation of motion having the form

$$3H^2 \equiv 3 \left(\frac{\dot{a}}{a} \right)^2 = \beta P_8 \left(\frac{\dot{a}}{a}, \frac{\ddot{a}}{a}, \frac{\dddot{a}}{a}, \frac{\ddddot{a}}{a} \right), \quad (6.2)$$

where P_8 is a *polynomial* with respect to its arguments,

$$P_8 = \sum_{\substack{n_1+2n_2+3n_3+4n_4=8, \\ n_1, n_2, n_3, n_4 \geq 0}} c_{n_1 n_2 n_3 n_4} \left(\frac{\dot{a}}{a} \right)^{n_1} \left(\frac{\ddot{a}}{a} \right)^{n_2} \left(\frac{\dddot{a}}{a} \right)^{n_3} \left(\frac{\ddddot{a}}{a} \right)^{n_4} \quad (6.3)$$

⁹The proposal [37] to identify the inflaton with the SM Higgs boson requires its non-minimal coupling to gravity, which does not fit to string theory.

Here the sum goes over the *integer* partitions $(n_1, 2n_2, 3n_3, 4n_4)$ of 8, the dots stand for the derivatives with respect to time t , and $c_{n_1 n_2 n_3 n_4}$ are some real coefficients. The highest derivative enters linearly at most, $n_4 = 0, 1$.

The FRW Ansatz with $k = 0$ gives the following non-vanishing curvatures:

$$R^0_{\mu 0 \nu} = \delta_{\mu \nu} \ddot{a} \dot{a}, \quad R^\mu_{\nu \lambda \rho} = (\delta^\mu_\lambda \delta_{\nu \rho} - \delta^\mu_\rho \delta_{\nu \lambda}) (\dot{a})^2, \quad R^\mu_\nu = -\delta^\mu_\nu \left[\frac{\ddot{a}}{a} + 2 \left(\frac{\dot{a}}{a} \right)^2 \right] \quad (6.4)$$

where $\mu, \nu, \lambda, \rho = 1, 2, 3$. For example, in the case of the $(BR)^2$ gravity (3.8), after a straightforward (though quite tedious) calculation of the mixed 00-equation without matter and with the curvatures (6.4), we find

$$3H^2 + \beta \left[9 \left(\frac{\ddot{a}}{a} \right)^4 - 36H^2 \left(\frac{\ddot{a}}{a} \right)^3 + 84H^4 \left(\frac{\ddot{a}}{a} \right)^2 - 36H \left(\frac{\ddot{a}}{a} \right)^2 \left(\frac{\ddot{\ddot{a}}}{a} \right) + 63H^8 - 72H^3 \left(\frac{\ddot{a}}{a} \right) \left(\frac{\ddot{\ddot{a}}}{a} \right) + 48H^6 \left(\frac{\ddot{a}}{a} \right) - 24H^5 \left(\frac{\ddot{a}}{a} \right) \right] = 0 \quad (6.5)$$

It is remarkable that the 4th order time derivatives (present in various terms of eq. (3.8)) cancel, whereas the square of the 3rd order time derivative of the scale factor, $\ddot{\ddot{a}}^2$, does not appear at all in this equation.¹⁰

Our generalized Friedmann equation (6.2) applies to *any* combination of the quartic curvature terms in the action, including the Ricci-dependent terms. The coefficients $c_{n_1 n_2 n_3 n_4}$ in eq. (6.3) can be thought of as linear combinations of the coefficients in the most general quartic curvature action. The polynomial (6.3) merely has 12 undetermined coefficients, that is considerably less than a 100 of the coefficients in the most general quartic curvature action.

The structure of eqs. (6.2) and (6.3) admits the existence of rather generic exact inflationary solutions without a spacetime singularity. Indeed, when using the most naive (de Sitter) Ansatz for the scale factor,

$$a(t) = a_0 e^{Bt} \quad (6.6)$$

with some real positive constants a_0 and B , and substituting eq. (6.6) into eq. (6.2), we get $3B^2 = (\#)\beta B^8$, whose coefficient $(\#)$ is just a sum of all

¹⁰Taking Weyl tensors instead of Riemann curvatures leads to the vanishing coefficients.

c -coefficients in eq. (6.3). Assuming the $(\#)$ to be positive, we find an exact solution,

$$B = \left(\frac{3}{\#\beta} \right)^{1/6} \quad (6.7)$$

This solution is non-perturbative in β , i.e. it is impossible to get it when considering the quartic curvature terms as a perturbation. Of course, the assumption that we are dealing with the leading correction, implies $Bt \ll 1$. Because of eqs. (2.11) and (6.7), it leads to the natural hierarchy

$$\kappa M_{\text{KK}} \ll 1 \quad \text{or} \quad l_{\text{Pl}} \ll l_{\text{KK}} \quad (6.8)$$

where we have introduced the four-dimensional Planck scale $l_{\text{Pl}} = \kappa$ and the compactification scale $l_{\text{KK}} = M_{\text{KK}}^{-1}$.

The *effective* Hubble scale B of eq. (6.7) should be lower than the *effective* (with warping) KK scale $M_{\text{KK}}^{\text{eff.}} = e^A M_{\text{KK}}$, in order to validate our four-dimensional description of gravity, i.e. the ignorance of all KK modes,

$$B < M_{\text{KK}}^{\text{eff.}} \quad (6.9)$$

It rules out the naive KK reduction (with $A = 0$) but still allows the warped compactification (2.6), when the average warp factor is tuned,

$$e^A < \frac{(\kappa M_{\text{KK}})^{2/5}}{(9/\#)^{3/10} 2^{23/10} \pi} \sim \mathcal{O}(10^{-3}) \quad (6.10)$$

where we have used eq. (2.11) and have estimated $(\#)$ by order 10.

The exact solution (6.6) is non-singular, while it describes an inflationary isotropic and homogeneous early universe.¹¹ Given the expanding universe, the curvatures decrease, so that the higher curvature terms cease to be the dominant contributions against the matter terms we ignored in the equations of motion. The matter terms may provide a mechanism for ending the geometrical inflation and reheating (i.e. a Graceful Exit to the standard cosmology).

To be truly inflationary solutions, eqs. (6.6) and (6.7) should correspond to the stable fixed points (or attractors) [1]. The stability conditions are easily derived along the standard lines (see e.g., refs. [41, 42]). When using the parametrization

$$a(t) = e^{\lambda(t)} \quad , \quad (6.11)$$

¹¹The exact de Sitter solutions in the special case (2.8) were also found in ref. [40].

we easily find

$$\begin{aligned}
\frac{\dot{a}}{a} &= \dot{\lambda} \quad , \\
\frac{\ddot{a}}{a} &= \ddot{\lambda} + (\dot{\lambda})^2 \quad , \\
\frac{\dddot{a}}{a} &= \dddot{\lambda} + 3 \ddot{\lambda} \dot{\lambda} + (\dot{\lambda})^3 \quad , \\
\frac{\dots{a}}{a} &= \dots{\lambda} + 4 \ddot{\lambda} \dot{\lambda} + 6 \ddot{\lambda} (\dot{\lambda})^2 + 3 (\ddot{\lambda})^2 + (\dot{\lambda})^4
\end{aligned} \tag{6.12}$$

Equation (6.3) now takes the form

$$P_8 = \sum_{\substack{n_1+2n_2+3n_3+4n_4=8, \\ n_1, n_2, n_3, n_4 \geq 0}} d_{n_1 n_2 n_3 n_4} \left(\dot{\lambda} \right)^{n_1} \left(\ddot{\lambda} \right)^{n_2} \left(\dddot{\lambda} \right)^{n_3} \left(\dots{\lambda} \right)^{n_4} \tag{6.13}$$

where the d -coefficients are linear combinations of the c -coefficients (easy to find). Equations (6.6) and (6.7) are also simplified,

$$\lambda(t) = Bt + \lambda_0 \quad , \quad \text{where} \quad a_0 = e^{\lambda_0} \quad \text{and} \quad d_{8000} = \# \quad . \tag{6.14}$$

The solution (6.14) can be considered as the fixed point of the equations of motion (6.2) in a generic case,

$$3y_1^2 = \beta P_8(y_1, y_2, y_3, \dot{y}_3) \equiv \beta P_{8,0}(y_1, y_2, y_3) + \beta P_4(y_1, y_2, y_3) \dot{y}_3 \quad , \tag{6.15}$$

where we have introduced the notation

$$y_1 = \dot{\lambda} \quad , \quad y_2 = \ddot{\lambda} \quad , \quad y_3 = \ddot{\lambda} \quad . \tag{6.16}$$

Equation (6.15) can be brought into an autonomous form,

$$\begin{aligned}
\dot{y}_1 &= y_2 \quad , \\
\dot{y}_2 &= y_3 \quad , \\
\dot{y}_3 &= \frac{3y_1^2 - \beta P_{8,0}(y_1, y_2, y_3)}{\beta P_4(y_1, y_2, y_3)} \equiv f(y_1, y_2, y_3)
\end{aligned} \tag{6.17}$$

that is quite suitable for the stability analysis against small perturbations about the fixed points, $y_a = y_a^{\text{fixed}} + \delta y_a$, where $a = 1, 2, 3$. We find

$$\begin{aligned}\delta \dot{y}_1 &= \delta y_2 \ , \\ \delta \dot{y}_2 &= \delta y_3 \ , \\ \delta \dot{y}_3 &= \left. \frac{\partial f}{\partial y_1} \right| \delta y_1 + \left. \frac{\partial f}{\partial y_2} \right| \delta y_2 + \left. \frac{\partial f}{\partial y_3} \right| \delta y_3 \ ,\end{aligned}\tag{6.18}$$

where all the partial derivatives are taken at the fixed point (denoted by $|$). The fixed points are stable when all the eigenvalues of the matrix

$$\hat{M} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \left. \frac{\partial f}{\partial y_1} \right| & \left. \frac{\partial f}{\partial y_2} \right| & \left. \frac{\partial f}{\partial y_3} \right| \end{pmatrix}\tag{6.19}$$

in eq. (6.18) are *negative* or have negative real parts [41, 42]. Then the fixed point is a stable attractor.

To the end of this Section, we would like to investigate how the symmetries of string theory are going to affect the coefficients of our generalized Friedmann equation. Here we apply the scale factor duality [39] by requiring our equation (6.2) to be invariant under the duality transformation

$$a(t) \leftrightarrow \frac{1}{a(t)} \equiv b(t)\tag{6.20}$$

This duality is a cosmological version of the genuine stringy T-duality (which is the symmetry of the non-perturbative string spectrum), in the case of time-dependent backgrounds. The scale factor duality is merely the symmetry of the (perturbative) equations of motion of the background fields. It is used e.g. in the so-called pre-big bang scenario [43], in order to avoid the cosmological singularity.

In the λ -parametrization (6.11) the duality transformation (6.20) takes the very simple form

$$\lambda(t) \leftrightarrow -\lambda(t)\tag{6.21}$$

The equations of motion in the form (6.15) are manifestly invariant under $\lambda(t) \rightarrow \lambda(t) + \lambda_0$, where λ_0 is an arbitrary constant.

It is straightforward to calculate how the right-hand-side of eq. (6.2) transforms under the duality (6.20) by differentiating eq. (6.20). We find

$$\begin{aligned}
\frac{\dot{a}}{a} &= -\frac{\dot{b}}{b} , \quad \frac{\ddot{a}}{a} = -\frac{\ddot{b}}{b} + 2 \left(\frac{\dot{b}}{b} \right)^2 , \\
\frac{\dddot{a}}{a} &= -\frac{\dddot{b}}{b} + 6 \left(\frac{\dot{b}}{b} \right) \left(\frac{\ddot{b}}{b} \right) - 6 \left(\frac{\dot{b}}{b} \right)^3 , \\
\frac{\dots{a}}{a} &= -\frac{\dots{b}}{b} + 6 \left(\frac{\ddot{b}}{b} \right)^2 + 8 \left(\frac{\dot{b}}{b} \right) \left(\frac{\dddot{b}}{b} \right) - 36 \left(\frac{\dot{b}}{b} \right)^2 \left(\frac{\ddot{b}}{b} \right) + 24 \left(\frac{\dot{b}}{b} \right)^4
\end{aligned} \tag{6.22}$$

To see how the duality affects the polynomial P_8 , we consider the case with the 3rd order time derivatives, motivated by eq. (6.5). We introduce the notation

$$\frac{\dot{a}}{a} = x , \quad \frac{\ddot{a}}{a} = y , \quad \frac{\dddot{a}}{a} = z \tag{6.23}$$

so that the duality invariance condition reads

$$P_8(-x, 2x^2 - y, 6xy - 6x^3 - z) = P_8(x, y, z) \tag{6.24}$$

The structure of the polynomial P_8 in eq. (6.3), as the sum over partitions of 8, restricts a solution to eq. (6.24) to be most quadratic in z ,

$$P_8(x, y, z) = a_2(x, y)z^2 + b_5(x, y)z + c_8(x, y) \tag{6.25}$$

whose coefficients are polynomials in (x, y) , of the order being given by their subscripts, i.e.

$$\begin{aligned}
a_2(x, y) &= a_0x^2 + a_1y , \\
b_5(x, y) &= b_0x^5 + b_1x^3y + b_2xy^2 , \\
c_8(x, y) &= c_4y^4 + c_3y^3x^2 + c_2y^2x^4 + c_1yx^6 + c_0x^8
\end{aligned} \tag{6.26}$$

After a substitution of eqs. (6.25) and (6.14) into eq. (6.24), we get an *overdetermined* system of linear equations on the coefficients. Nevertheless, we find that there is a consistent general solution,

$$\begin{aligned}
P_8(x, y, z) &= a_0x^2z^2 + (b_0x^5 - 3a_0xy^2)z \\
&\quad + c_4y^4 + (9a_0 - 4c_4)y^3x^2 + c_2y^23x^4 \\
&\quad + (8c_4 - 18a_0 - 3b_0 - 2c_2)yx^6 + c_0x^8
\end{aligned} \tag{6.27}$$

parameterized by merely five real coefficients $(a_0, b_0, c_4, c_2, c_0)$. Requiring the existence of the exact solution (6.6), i.e. the positivity of $(\#)$ in eq. (6.7), yields

$$5c_4 + c_0 > 11a_0 + 2b_0 + c_2 \quad (6.28)$$

As regards the $(BR)^2$ gravity representing the ‘minimal’ candidate for the off-shell superstring effective action, we checked that neither the duality invariant structure (6.15) nor the inequality (6.28) are satisfied by the coefficients present in eq. (6.5). We interpret it as the clear indications that some additional Ricci-dependent terms *have to be added* to the $(BR)^2$ terms or, equivalently, the $(BR)^2$ gravity is ruled out as the off-shell effective action for superstrings.

Finally, we would like to mention about some possible simplifications and generalizations.

The last equation (6.4) apparently implies that the Ricci-dependent terms in P_8 should have the factor of $(y + 2x^2)$. Hence, it may be possible to completely eliminate both the 4th and 3rd order time derivatives in our generalized Friedmann equations, though we are not sure that this choice is fully consistent. However, if so, instead of eq. (6.24) we would get another duality condition,

$$P_8(-x, 2x^2 - y) = P_8(x, y) \quad (6.29)$$

whose most general solution is simpler,

$$P_8(x, y) = c_0 x^8 + c_5 y(y - 2x^2) [y(y - 2x^2) - 4x^6] + c_6 x^4 y(y - 2x^2) \quad (6.30)$$

with merely three, yet to be determined coefficients (c_0, c_5, c_6) .

We would like to emphasize that our results above can be generalized to any finite order with respect to the spacetime curvatures in the off-shell superstring effective action, because it amounts to increasing the order of the polynomial P . The list (6.10) can be continued to any higher order in the derivatives. We can now speculate about the form of the generalized Friedmann equation to *all* orders in the curvature. It depends upon whether (i) there will be some finite maximal order of the time derivatives there, or (ii) the time derivatives of arbitrarily high order appear (we do not know about it). Given the case (i), we just drop the requirement that the right-hand-side of our cosmological equation (6.2) is a polynomial, and take a duality-invariant *function* P instead. In the case (ii), we should replace the

function by a *functional*, thus getting a non-local equation having the form

$$H^2 = \frac{\dot{a}^2}{a^2} = \beta P[a(t)] \quad (6.31)$$

whose functional P is subject to the non-trivial duality constraint

$$P[a(t)] = P[1/a(t)] . \quad (6.32)$$

Imposing *simultaneously* both conditions of stability and duality invariance leads to severe constraints on the c -coefficients. Hence, it also severely restricts the quantum ambiguities in the superstring-generated quartic curvature gravity. Finding their solutions seems to be a non-trivial mathematical problem. We would like to investigate it elsewhere [44].

7 Conclusion

The higher curvature terms in the gravitational action defy the famous Hawking-Penrose theorem [45] about the existence of a spacetime singularity in any exact solution to the Einstein equations. As we demonstrated in this paper, the initial cosmological singularity can be easily avoided by considering the superstring-motivated higher curvature terms on equal footing (i.e. non-perturbatively) with the Einstein-Hilbert term.

Our results predict the possible existence of the very short de Sitter phase driven by the quartic curvature terms, in the early inflationary epoch.

Though we showed the natural existence of inflationary (de Sitter) exact solutions without a spacetime singularity under rather generic conditions on the coefficients in the higher-derivative terms, it is not enough for robust physical applications. As a matter of fact, we assumed the dominance of the higher curvature gravitational terms over all matter contributions in the very early Universe at the Planck scale. However, given the expansion of the Universe under the geometrical inflation, the spacetime curvatures should decrease, so that the matter terms can no longer be ignored. The latter may effectively replace the geometrical inflation by another matter-dominated mechanism, thus allowing the inflation to continue substantially below the Planck scale.

In addition, the number of e-foldings (5.5) is just about one in our scenario based on the quartic curvature terms, which makes it difficult to compete

with the conventional inflation mechanisms [1]. An investigation of the possible ‘Graceful Exit’ strategies, towards a matter-driven inflation is, however, beyond the scope of the given paper.

The quartic curvature terms are also relevant to the Brandenberger-Vafa cosmological scenario of string gas cosmology [46] — see e.g. ref. [42] for a recent investigation of the higher curvature corrections there.¹²

The higher time derivatives in the equations of motion may be unavoidable when using the higher curvature terms, but we do not see that they constitute a trouble.

Gravity with the quartic curvature terms is a good playground for going beyond the Einstein equations. Our analysis may be part of a more general approach based on superstrings, including dynamical moduli and extra dimensions.

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¹²The higher curvature terms were considered only perturbatively in ref. [42].

8 Appendix A: our notation, and identities

We use the basic notation of ref. [20] with the signature $(+, -, -, -)$. The (Riemann-Christoffel) curvature tensor is given by

$$R^i{}_{klm} = \frac{\partial \Gamma^i_{km}}{\partial x^l} - \frac{\partial \Gamma^i_{kl}}{\partial x^m} + \Gamma^i_{nl} \Gamma^n_{km} - \Gamma^i_{nm} \Gamma^n_{kl} \quad (8.1)$$

in terms of the Christoffel symbols

$$\Gamma^i_{kl} = \frac{1}{2} g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right) \quad (8.2)$$

It follows

$$R_{iklm} = \frac{1}{2} \left(\frac{\partial^2 g_{im}}{\partial x^k \partial x^l} + \frac{\partial^2 g_{kl}}{\partial x^i \partial x^m} - \frac{\partial^2 g_{il}}{\partial x^k \partial x^m} - \frac{\partial^2 g_{km}}{\partial x^i \partial x^l} \right) + g_{np} (\Gamma^n_{kl} \Gamma^p_{im} - \Gamma^n_{km} \Gamma^p_{il}) \quad (8.3)$$

The traceless part of the curvature tensor is given by a Weyl tensor,

$$C_{ijkl} = R_{ijkl} - \frac{1}{2} (g_{ik} R_{jl} - g_{jk} R_{il} - g_{il} R_{jk} + g_{jl} R_{ik}) + \frac{1}{6} (g_{ik} g_{jl} - g_{jk} g_{il}) R \quad (8.4)$$

where we have introduced the Ricci tensor and the scalar curvature,

$$R_{ik} = g^{lm} R_{limk}, \quad R = g^{ik} R_{ik} \quad (8.5)$$

The dual curvature is defined by

$${}^* R_{iklm} = \frac{1}{2} E_{ikpq} R^{pq}{}_{lm} \quad (8.6)$$

where $E_{iklm} = \sqrt{-g} \varepsilon_{iklm}$ is Levi-Civita tensor.

The *Euler* (E) and *Pontryagin* (P) topological densities in four dimensions are

$$E_4 = \frac{1}{4} \varepsilon_{ijkl} \varepsilon^{mnpq} R^{ij}{}_{mn} R^{kl}{}_{pq} = {}^* R_{ijkl} {}^* R^{ijkl} \quad (8.7)$$

and

$$P_4 = {}^* R_{ijkl} R^{ijkl} \quad (8.8)$$

respectively.

The *Bel-Robinson* (BR) tensor is defined by [29]

$$\begin{aligned} T_R^{iklm} &= R^{ipql} R^k{}_{pq}{}^m + {}^* R^{ipql} {}^* R^k{}_{pq}{}^m \\ &= R^{ipql} R^k{}_{pq}{}^m + R^{ipqm} R^k{}_{pq}{}^l - \frac{1}{2} g^{ik} R^{pqr l} R_{pqr}{}^m \end{aligned} \quad (8.9)$$

Its Weyl cousin is given by

$$T_C^{iklm} = C^{ipql} C_{pq}^{k\ m} + C^{ipqm} C_{pq}^{k\ l} - \frac{1}{2} g^{ik} C^{pqlr} C_{pqr}^m \quad (8.10)$$

The Riemann-Christoffel curvature (modulo Ricci-dependent terms) is most easily described in the Petrov formalism [47] by imposing the Ricci-flatness condition $R_{ik} = 0$. A metric g_{mn} at a given point in space-time can always be brought into Minkowski form $\eta = \text{diag}(+, -, -, -)$, whereas the curvature tensor components can be represented by ¹³

$$A_{\alpha\beta} = R_{0\alpha 0\beta}, \quad C_{\alpha\beta} = \frac{1}{4} \epsilon_{\alpha\gamma\delta} \epsilon_{\beta\lambda\mu} R_{\gamma\delta\lambda\mu}, \quad B_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\gamma\delta} R_{0\beta\gamma\delta} \quad (8.11)$$

where the 3d tensors A and C are symmetric by definition, $\alpha, \beta, \dots = 1, 2, 3$, and $\epsilon_{\alpha\beta\gamma}$ is 3d Levi-Civita symbol normalized by $\epsilon_{123} = 1$.

The Ricci-flatness condition implies that A is traceless, B is symmetric, and $C = -A$. It is now natural to introduce a symmetric (traceless) complex 3d tensor

$$D_{\alpha\beta} = A_{\alpha\beta} + iB_{\alpha\beta} \quad (8.12)$$

and bring it into one of its canonical (Petrov) forms, called I, II or III, depending upon a number (3, 2 or 1, respectively) of eigenvectors of D . For our purposes, it is most convenient to use the form I with three independent (complex) eigenvectors, so that the real matrices A and B can be simultaneously diagonalized as

$$\begin{aligned} A_{\alpha\beta} &= \text{diag}(\alpha', \beta', -\alpha' - \beta') , \\ B_{\alpha\beta} &= \text{diag}(\alpha'', \beta'', -\alpha'' - \beta'') \end{aligned} \quad (8.13)$$

in terms of their real eigenvalues.

It is straightforward to write down any Riemann-Christoffel curvature invariants as the polynomials of their eigenvalues (8.13) in the Petrov I form. It is especially useful for establishing various identities modulo Ricci-dependent terms. We find ¹⁴

$$R_{mnpq} R^{mnpq} = 16 (\alpha'^2 + \beta'^2 + \alpha'\beta' - \alpha''^2 - \beta''^2 - \alpha''\beta'') \quad (8.14)$$

¹³We use the lower-case greek letters to represent vector indices in three (flat) spatial dimensions.

¹⁴We are grateful to A. Morishita for his help with calculations.

and

$${}^*R_{mnpq}R^{mnpq} = 16(-2\alpha'\alpha'' - 2\beta'\beta'' - \alpha''\beta' - \alpha'\beta'') \quad (8.15)$$

so that

$$\begin{aligned} & (R_{mnpq}R^{mnpq})^2 + ({}^*R_{mnpq}R^{mnpq})^2 \\ &= 2^8 (\alpha'^4 + \beta'^4 + \alpha''^4 + \beta''^4 + 2\alpha'^2\alpha''^2 + 2\beta'^2\beta''^2 \\ & \quad + 2\alpha'^3\beta' + 2\alpha'\beta'^3 + 2\alpha''^3\beta'' + 2\alpha''\beta''^3 \\ & \quad + 2\alpha'^2\alpha''\beta'' + 2\alpha''\beta'^2\beta'' + 2\alpha'\alpha''^2\beta' + 2\alpha'\beta'\beta''^2 \\ & \quad + 3\alpha'^2\beta'^2 + 3\alpha''^2\beta''^2 - \alpha'^2\beta''^2 - \alpha''^2\beta'^2 + 8\alpha'\alpha''\beta'\beta'') \end{aligned} \quad (8.16)$$

Similarly one finds

$${}^*R_{mnpq}{}^*R^{mnpq} = -16(\alpha'^2 + \beta'^2 + \alpha'\beta' - \alpha''^2 - \beta''^2 - \alpha''\beta'') \quad (8.17)$$

For example, when being compared to eq. (8.14), it yields the identity

$${}^*R_{mnpq}{}^*R^{mnpq} = -R_{mnpq}R^{mnpq} + \mathcal{O}(R_{mn}) \quad (8.18)$$

As regards the superstring correction (2.8) in four dimensions, we find

$$\begin{aligned} J_R &= 8(\alpha'^4 + \beta'^4 + \alpha''^4 + \beta''^4 + 2\alpha'^2\alpha''^2 + 2\beta'^2\beta''^2 \\ & \quad + 2\alpha'^3\beta' + 2\alpha'\beta'^3 + 2\alpha''^3\beta'' + 2\alpha''\beta''^3 \\ & \quad + 2\alpha'^2\alpha''\beta'' + 2\alpha''\beta'^2\beta'' + 2\alpha'\alpha''^2\beta' + 2\alpha'\beta'\beta''^2 \\ & \quad + 3\alpha'^2\beta'^2 + 3\alpha''^2\beta''^2 - \alpha'^2\beta''^2 - \alpha''^2\beta'^2 + 8\alpha'\alpha''\beta'\beta'') \end{aligned} \quad (8.19)$$

The BR tensor (8.9) squared in the Petrov I form reads

$$\begin{aligned} T_{mnpq}T^{mnpq} &= 2^6 (\alpha'^4 + \beta'^4 + \alpha''^4 + \beta''^4 + 2\alpha'^2\alpha''^2 + 2\beta'^2\beta''^2 \\ & \quad + 2\alpha'^3\beta' + 2\alpha'\beta'^3 + 2\alpha''^3\beta'' + 2\alpha''\beta''^3 \\ & \quad + 2\alpha'^2\alpha''\beta'' + 2\alpha''\beta'^2\beta'' + 2\alpha'\alpha''^2\beta' + 2\alpha'\beta'\beta''^2 \\ & \quad + 3\alpha'^2\beta'^2 + 3\alpha''^2\beta''^2 - \alpha'^2\beta''^2 - \alpha''^2\beta'^2 + 8\alpha'\alpha''\beta'\beta'') \end{aligned} \quad (8.20)$$

As a result, we find the identities

$$T_{mnpq}T^{mnpq} = 8J_R = \frac{1}{4} [(R_{mnpq}R^{mnpq})^2 + ({}^*R_{mnpq}{}^*R^{mnpq})^2] \quad (8.21)$$

which are valid on-shell, i.e. modulo Ricci-tensor-dependent terms.

9 Appendix B: two-component formalism

To complete our notation, we summarize basic definitions and main features of the two-component spinor formalism in gravitation (*cf.* refs. [13, 16]). The main point is the use of an $sl(2; \mathbf{C})$ algebra isomorphic to the Lorentz algebra $so(1, 3; \mathbf{R})$.

We use lower-case (middle) latin indices for the curved space-time vector indices, capital (early) latin letters for the tangent (flat spacetime) vector indices, and lower-case (early) greek letters for the (tangent spacetime) spinor indices, $i, j, k, \dots = 0, 1, 2, 3$ and $A, B, C, \dots = 0, 1, 2, 3$, whereas $\alpha, \beta, \dots = 1, 2$ and $\dot{\alpha}, \dot{\beta}, \dots = \dot{1}, \dot{2}$.

A four-component Dirac spinor Ψ can be decomposed into its chiral and anti-chiral parts, ψ_α and $\bar{\psi}^{\dot{\beta}}$, by using the chiral projectors $\Gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$, where $\gamma_5^2 = 1$. The simplest form of chiral decomposition is obtained in the basis for Dirac gamma matrices with a diagonal $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ matrix,

$$\gamma^A = \begin{pmatrix} 0 & \sigma^A_{\alpha\dot{\beta}} \\ \tilde{\sigma}^{A\dot{\beta}\alpha} & 0 \end{pmatrix}, \quad \sigma^A = (\mathbf{1}, i\vec{\sigma}), \quad \tilde{\sigma}^A = (\mathbf{1}, -i\vec{\sigma}) \quad (9.1)$$

Here $\mathbf{1}$ is a unit 2×2 matrix, and $\vec{\sigma}$ are three Pauli matrices.

Given a vector field $V_i(x)$ in a curved spacetime, it can always be represented by a bispinor field $V_{\alpha\dot{\beta}}^{\dot{\alpha}\beta}(x)$,

$$V_{\alpha\dot{\beta}}^{\dot{\alpha}\beta} = V_i e_A^i \sigma^A_{\alpha\dot{\beta}} \quad , \quad V_i = e_i^B \frac{1}{2} V_{\alpha\dot{\beta}}^{\dot{\alpha}\beta} \tilde{\sigma}_B^{\beta\alpha} \quad (9.2)$$

where we have introduced the vierbein $e_A^i(x)$, together with its inverse $e_i^A(x)$, obeying the relations

$$g_{ij} e_A^i e_B^j = \eta_{AB} \quad , \quad \eta_{AB} e_i^A e_j^B = g_{ij} \quad (9.3)$$

For instance, one easily finds that the metric in the two-component formalism can be represented by a product of two Levi-Civita symbols,

$$g_{\alpha\dot{\beta}\dot{\alpha}\beta} = \varepsilon_{\alpha\dot{\beta}} \varepsilon_{\dot{\alpha}\beta} \quad (9.4)$$

As regards the curvature tensor, it can be naturally decomposed in the

two-component formalism as follows:

$$\begin{aligned}
R_{\alpha\beta\gamma\delta\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = & C_{\alpha\beta\gamma\delta}\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\delta}} + \bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} \\
& + D_{\dot{\alpha}\dot{\beta}\gamma\delta}\varepsilon_{\alpha\beta}\varepsilon_{\dot{\gamma}\dot{\delta}} + \bar{D}_{\alpha\beta\dot{\gamma}\dot{\delta}}\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\gamma\delta} \\
& + E\left(\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta} + \varepsilon_{\alpha\delta}\varepsilon_{\beta\gamma}\right)\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\delta}} + \bar{E}\left(\varepsilon_{\dot{\alpha}\dot{\gamma}}\varepsilon_{\dot{\beta}\dot{\delta}} + \varepsilon_{\dot{\alpha}\dot{\delta}}\varepsilon_{\dot{\beta}\dot{\gamma}}\right)\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta}
\end{aligned} \tag{9.5}$$

The four-spinor C (or \bar{C}) is *totally symmetric* with respect to its chiral (or anti-chiral) spinor indices, while it is also traceless, thus representing the self-dual (or anti-self-dual) part of the Weyl tensor (8.4),

$$C_{\alpha\beta\gamma\delta\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} = C_{\alpha\beta\gamma\delta}\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\dot{\gamma}\dot{\delta}} + \bar{C}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}\varepsilon_{\alpha\beta}\varepsilon_{\gamma\delta} \tag{9.6}$$

The four-spinor $D_{\dot{\alpha}\dot{\beta}\gamma\delta}$ is symmetric with respect to its first two indices, as well as with respect to the last two indices, while it is also traceless, thus representing the traceless part of the Ricci tensor,

$$R_{\alpha\gamma\dot{\alpha}\dot{\gamma}} = \varepsilon^{\delta\beta}\varepsilon^{\dot{\delta}\dot{\beta}}R_{\alpha\beta\gamma\delta\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \tag{9.7}$$

The scalar $E = \bar{E}$ represents the scalar curvature R . One easily finds

$$R_{\alpha\beta\dot{\alpha}\dot{\beta}} = -2D_{\dot{\alpha}\dot{\beta}\alpha\beta} + 6E\varepsilon_{\dot{\alpha}\dot{\beta}}\varepsilon_{\alpha\beta}, \quad R = 24E \tag{9.8}$$

The Bianchi II identities $\nabla_{[m}R_{ij]kl} = 0$ in the two-component formalism read as follows:

$$\nabla_{\dot{\beta}}^{\alpha}\dot{C}_{\alpha\beta\gamma\delta} = \nabla_{(\beta}^{\dot{\alpha}}D_{\gamma\delta)\dot{\alpha}\dot{\beta}}, \quad \nabla^{\gamma\dot{\alpha}}D_{\gamma\delta\dot{\alpha}\dot{\beta}} + 3\nabla_{\dot{\delta}\dot{\beta}}^{\dot{\alpha}}E = 0 \tag{9.9}$$

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